Equations for designing a flat plate device with a linear spring

Per the "Mechanical Energy Absorber" (U.S. Patent Number **6,006,874**) patent, we can replace Zener's nonlinear spring with a linear spring and realize several advantages. The following equations will help in designing a flat plate device with normal structural materials, instead of a modified piece of foam insulation. They cover everything Zener derived for the nonlinear case. To actually build a device, we'd have to add discussions of allowable plate deflections and the lateral dimensions required. We intend to add those soon.

A moving solid object of mass, m, having velocity, v_0, impacts a linear spring with spring rate, k, which is attached to the center of a thin flat plate of thickness, h.

y = displacement of the mass from position at initial contact

z = displacement of the center of the plate from position at initial contact

E = modulus of elasticity of plate material

ν = Poisson's ratio of plate material

ρ = density of plate material

x = y – z = deflection of spring

$$E' = E\big/\left(1-\nu^2\right); \quad c = 4h^2\sqrt{\frac{\rho E'}{3}}; \quad F = -m\frac{d^2y}{dt^2} = k(y-z) = c\frac{dz}{dt}$$

$$\frac{d^2x}{dt^2} + \frac{k}{c}\frac{dx}{dt} + \frac{k}{m}x = 0$$

$$d = \frac{2c}{\sqrt{km}}; \quad q = \sqrt{\frac{k}{m} - \left(\frac{k}{2c}\right)^2}$$

$$x = \frac{v_0}{q}e^{\frac{-kt}{2c}}\sin(qt); \quad t_f = \frac{\pi}{q}; \quad v_f = -v_0 e^{\frac{-\pi k}{2cq}}$$

$$rbh = relative\ bounce\ height = \frac{bounce\ height}{drop\ height} = \frac{final\ kinetic\ energy\ of\ mass}{initial\ kinetic\ energy\ of\ mass} = \left(\frac{v_f}{v_0}\right)^2 = e^{\frac{-\pi k}{cq}}$$

$$\therefore rbh = e^{\frac{-2\pi}{\sqrt{d^2-1}}}$$

$$T = \left(\frac{1}{q}\right)\tan^{-1}\sqrt{d^2-1}; \quad x_{\max} = v_0\sqrt{\frac{m}{k}}e^{\frac{-kT}{2c}}; \quad F_{\max} = kx_{\max}; \quad z_{\max} = \frac{mv_0}{c}\left(1+\sqrt{rbh}\right)$$